Maths Calculation Policy
September 2013
Introduction

At Ernehale Junior we believe that we should build upon the mental, oral and practical skills children should have developed at KS1. We are keen to develop speaking and listening processes which allow children to understand mathematical concepts and can articulate them clearly to a variety of audiences including their peers and adults they are working with. We want the children to be able to make the jump and links between mental and more informal written methods. As children’s mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

We have put a greater emphasis on number and the use and understanding of place value as this is key to all areas of number and vital to ensuring a clear understanding of what number is and can do. We also feel that despite the new NC not placing as much emphasis on using and apply that this is still a key tool for our children to develop mathematically, and so this will still have a place in our teaching and learning of mathematics along with a focus on speaking and listening to develop mathematical processes orally in order to explain, reason and justify processes and understanding in maths particularly in number.

This links to what is stated in the NC

**Become fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems

**Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

**Can solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The overall aim is that when children leave Ernehale Junior is that they:

- have a secure knowledge of number facts including place value and a assured clear understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.
• To be able to access the KS 3 curriculum if appropriate to ensure all children are
challenged and stimulated.
• To have an extensive vocabulary that allows them to articulate, reason and justify
processes and calculations.
• To be able to apply knowledge learnt in a variety of different situations, combining different
operations if necessary.

**Mental methods of calculation**

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral
and mental work must lay the foundations by providing children with a good understanding of how
the four operations build on efficient counting strategies and a secure knowledge of place value and
number facts. Later work must ensure that children recognise how the operations relate to one
another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and
mental work provides practice and consolidation of these ideas. It must give children the opportunity
to apply what they have learned to particular cases, exemplifying how the rules and laws work, and
to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be
maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of
structured practice and repetition. It requires an understanding of number patterns and relationships
developed through directed enquiry, use of models and images and the application of acquired
number knowledge and skills. Secure mental calculation requires the ability to:
• recall key number facts instantly – for example, all addition and subtraction facts for each
number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and
multiplication facts up to 10 × 10 (Year 4);

**Written methods of calculation**

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient
written method for each operation with confidence and understanding. This guidance promotes the use
of what are commonly known as 'standard' written methods – methods that are efficient and work for any
calculations, including those that involve whole numbers or decimals. They are compact and
consequently help children to keep track of their recorded steps. Being able to use these written methods
gives children an efficient set of tools they can use when they are unable to carry out the calculation in
their heads or do not have access to a calculator. We want children to know that they have such a
reliable, written method to which they can turn when the need arises. These methods do not have to be
the same for every child and the children should be given opportunities to use a variety methods.

In setting out these aims, the intention is that we adopt greater consistency in our approach to
calculation. The challenge is for our teachers to determine when their children should move on to
a refinement in the method and become confident and more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental, written or calculator
method based on the knowledge that they are in control of this choice as they are able to carry
out all three methods with confidence.

**Talking:**

• There is evidence that peer interactions are a main facilitator factor for socio-
cognitive development and their performance in maths tasks.
• A classroom culture of questioning in which pupils learn from shared discussions
with teachers and peers.
• Teachers role helps pupils to feel free and confident to comment and to suggest
strategies even if there are errors in calculations. This allows for all pupils to reflect and
suggest more appropriate methods.
• To encourage teachers to value all contributions and be willing to change their own minds
in the light of what the pupil says.
• Tasks are planned so there are opportunities for pupils to communicate their
We value the communication between teachers and pupils, pupils and their peers. When children feel confident to communicate their ideas and discuss their findings openly this improves their level of understanding. It has been proved that children will remember 70% of what they have been learning if they taken an active part in the lesson compared to a passive learner who will only retain 20% of what has been taught.

**Vocabulary**

Vocabulary builds as the children progress through the school and the vocabulary highlighted for each year group are the most frequently used words. Refer to new NC for this.

**Choosing the appropriate strategy**

Recording in mathematics, and in calculation in particular is an important tool both for furthering the understanding of ideas and for communicating those ideas to others. A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important children acquire secure mental methods of calculation and one efficient written method of calculation for addition, subtraction, multiplication and division which they know they can rely on when mental methods are not appropriate. As a long term aim children should be able to choose an efficient method; mental, written, calculator- that is appropriate to a given task.
Written methods for addition

These phases show the building up to using an efficient written method for addition of whole numbers.

To add successfully, children need to be able to:

• recall all addition pairs to 9 + 9 and complements in 10 (making ten e.g. 4+6=10);
• add mentally a series of one-digit numbers, such as 5 + 8 + 4;
• add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
• partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

<table>
<thead>
<tr>
<th>Phase 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Develop secure one-one correspondence and understanding of addition.</td>
</tr>
</tbody>
</table>

- **Count accurately 0-10**
- **Recognise and write numerals 1-10**

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>?</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>?</td>
<td>5</td>
<td>?</td>
<td>7</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>?</td>
<td>5</td>
<td>?</td>
<td>10</td>
</tr>
</tbody>
</table>

What is the number before 5? And after 5?
Before 10! What is the number between 3 and 5?
What numbers are between 7 and 10?

- Count and add together sets of real objects and pictures.

3+2 = 5

❤️❤️❤️ + ❤️❤️❤️❤️
Phase 2: The number line and 100 square

The number line helps children to move from using concrete objects.

Children begin to split a number to add to the nearest multiple of 10 and then count on.

The 100 square supports children's understanding when adding ten to any number the units stay the same and the tens go up. Eventually this will be done mentally.

Phase 2

To use a number line to add one or more numbers together.

\[ 8 + 1 = 9 \]

- To be able to add through 10.

\[ 8 + 5 = 13 \]

100 Square

To be able to add 10 to any number up to 100 using a 100 square and then count on.

\[ 9 + 10 = 10 + 9 \]

= 19

To be able to add multiples of 10 to any number up to 100 using a 100 square.

\[ 34 + 40 = 74 \] (100 square picture?)

Phase 3: The empty number line

- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

Phase 3

Steps in addition can be recorded on an empty number line. The steps often bridge through a multiple of 10.

\[ 8 + 7 = 15 \]

To be able to add 2 two digit numbers on an empty number line.

\[ 48 + 36 = 84 \]

or:

\[ 48 + 50 + 34 = 132 \]

As above

Key vocabulary

Add, more, count on, plus, sum, total, altogether, partition, how many. Multiple of 10, number line, 100 square
### Phase 4: Partitioning
- The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and units mirrors the column method where units are placed under units and tens under tens. This also links to mental methods.

<table>
<thead>
<tr>
<th>Example</th>
<th>47 + 76</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 117 + 6</td>
<td></td>
</tr>
<tr>
<td>= 123</td>
<td></td>
</tr>
</tbody>
</table>

or

<table>
<thead>
<tr>
<th>Example</th>
<th>47 + 76</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 + 70 = 110</td>
<td></td>
</tr>
<tr>
<td>7 + 6 = 13</td>
<td></td>
</tr>
<tr>
<td>= 123</td>
<td></td>
</tr>
</tbody>
</table>

Partitioned numbers are then written under one another:

<table>
<thead>
<tr>
<th>Example</th>
<th>47 + 76</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 + 13 = 123</td>
<td></td>
</tr>
</tbody>
</table>

### Phase 5: Expanded method in columns
- Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sums either the tens or the units can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the units digits first always.
- The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

### Phase 6: Column method
- In this method, recording is reduced further. Carry digits are recorded below the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.
- Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

<table>
<thead>
<tr>
<th>Example</th>
<th>47 + 258 + 366</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 76</td>
<td>+ 87 + 458</td>
</tr>
<tr>
<td>123</td>
<td>345</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.
Phase 6: Column method expanded to decimals
In this phase the standard method is expanded to include the use of decimals.

<table>
<thead>
<tr>
<th>Phase 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>45.24</td>
</tr>
<tr>
<td>+ 26.59</td>
</tr>
<tr>
<td>71.83</td>
</tr>
<tr>
<td>1 1</td>
</tr>
</tbody>
</table>

As above
Units boundary, tenths boundary, hundredths boundary.

Written methods for subtraction

These Phases show the building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

**Note:** It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

**Phase 1**

Develop secure one-one correspondence and understanding of subtraction.

- Count accurately forwards and backwards from 0-10
- Recognise and write numerals 1-10

- 1 2 3 4 5 6 7 8 9 10

What is the number before 5? And after 5?
Before 10! What is the number between 3 and 5?
What numbers are between 7 and 10?

- Count and subtract sets of real objects and pictures.
  5-2 = 3
### Phase 2 – number line and 100 square

**Phase 2**
To be able to subtract/take away one less on a number line.

\[
\begin{array}{c}
8 - 1 = 7 \\
\hline
\end{array}
\]

![Number Line](image)

To be able to subtract through 10

\[
\begin{array}{c}
13 - 5 = 8 \\
\hline
\end{array}
\]

![Number Line](image)

To be able to subtract 10 from any number up to 100 using a 100 square.

\[
34 - 10 = 24
\]

To be able to subtract multiples of 10 from any number up to 100 using a 100 square.

\[
84 - 40 = 44 \text{ (100 square)}
\]

### Phase 3: Using the empty number line

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.

- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

**Phase 3**
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

\[
15 - 7 = 8
\]

![Number Line](image)

74 – 27 = 47 worked by counting back:

\[
\begin{array}{c}
47 - 3 = 44 \\
44 + 20 = 64 \\
64 - 4 = 60 \\
60 + 7 = 67 \\
67 - 3 = 64 \\
64 + 10 = 74
\end{array}
\]

The steps may be recorded in a different order:

\[
\begin{array}{c}
47 - 20 = 27 \\
27 - 3 = 24 \\
24 - 4 = 20
\end{array}
\]

or combined:

\[
\begin{array}{c}
47 - 23 = 24 \\
24 - 4 = 20
\end{array}
\]

Subtract a 2 two digit number by finding the difference through counting on an empty number line.

\[
45 - 23 =
\]

\[
\begin{array}{c}
23 + 7 = 30 \\
30 + 10 = 40 \\
40 + 5 = 45
\end{array}
\]

10 + 7 + 5 = 22
### Phase 4: Partitioning

- Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For $74 - 27$ this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

- This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.

![Partitioning Diagram](image)

### Stage 5: Expanded layout, leading to column method

- Partitioning the numbers into tens and units and writing one under the other mirrors the column method, where units are placed under units and tens under tens.

- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.

- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

### Example: $74 - 27$

<table>
<thead>
<tr>
<th></th>
<th>60</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>20</td>
<td>+ 7</td>
</tr>
<tr>
<td>=</td>
<td>70</td>
<td>+ 4</td>
</tr>
</tbody>
</table>

### Example: $741 - 367$

<table>
<thead>
<tr>
<th></th>
<th>600</th>
<th>130</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>300</td>
<td>+ 60</td>
<td>+ 7</td>
</tr>
<tr>
<td>=</td>
<td>400</td>
<td>+ 70</td>
<td>+ 4</td>
</tr>
</tbody>
</table>

### Example: $54 - 7$

- $74 - 27 = 74 - 20 - 7$
- $= 54 - 7$
- $= 47$

This method relies on secure mental skills.
### The expanded method for three-digit numbers

Example: 563 − 241, no adjustment or decomposition needed

<table>
<thead>
<tr>
<th>Expanded method</th>
<th>Leading to</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 + 60 + 3</td>
<td>563</td>
</tr>
<tr>
<td>− 200 + 40 + 1</td>
<td>− 241</td>
</tr>
<tr>
<td>300 + 20 + 2</td>
<td>322</td>
</tr>
</tbody>
</table>

Start by subtracting the units, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying ‘sixty take away forty’, not ‘six take away four’.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

\[
\begin{align*}
500 + 60 + 3 & \quad 400 + 160 + 3 & \quad 500 + 60 + 3 & \quad 400 + 160 + 3 \quad -5 \quad -63 \\
-200 + 70 + 1 & \quad -200 + 70 + 1 & \quad -200 + 70 + 1 & \quad -27 \quad 1 \\
200 + 90 + 2 & \quad 200 + 90 + 2 & \quad 29 \quad 2 \\
\end{align*}
\]

Begin by reading aloud the number from which we are subtracting: ‘five hundred and sixty-three’. Then discuss the hundreds, tens and units components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes ‘160 minus 70’, an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the units

\[
\begin{align*}
500 + 60 + 3 & \quad 400 + 150 + 13 & \quad 500 + 60 + 4 & \quad 4 \quad 15 \quad 13 \\
-200 + 70 + 8 & \quad -200 + 70 + 8 & \quad -200 + 70 + 8 & \quad -27 \quad 8 \\
200 + 80 + 5 & \quad 200 + 80 + 5 & \quad 28 \quad 5 \\
\end{align*}
\]

Here both the tens and the units digits to be subtracted are bigger than both the tens and the units digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

\[
\begin{align*}
500 + 0 + 3 & \quad 400 + 90 + 13 & \quad 500 + 90 + 3 & \quad 4 \quad 9 \quad 13 \\
-200 + 70 + 8 & \quad -200 + 70 + 8 & \quad -200 + 70 + 8 & \quad -27 \quad 8 \\
200 + 20 + 5 & \quad 200 + 20 + 5 & \quad 22 \quad 5 \\
\end{align*}
\]

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

**Teaching point:**

*Ensure that the pupils are secure at the expanded stage before progressing to the standard decomposition method*

Our children are encouraged to use decomposition when they have a secure understanding of place value in relation to subtraction.

473

- 389

84
Written methods for multiplication of whole numbers

These phases show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:
• recall all multiplication facts to $10 \times 10$;
• partition number into multiples of one hundred, ten and one;
• work out products such as $70 \times 5$, $70 \times 50$, $700 \times 5$ or $700 \times 50$ using the related fact $7 \times 5$ and their knowledge of place value;
• add two or more single-digit numbers mentally;
• add multiples of 10 (such as $60 + 70$) or of 100 (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
• add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.
Phase 1: Hands on experiences

Initially children put objects into groups/sets.

Solve multiplication through repeated addition.

To use arrays to solve multiplication problems.

Children begin to identify patterns within multiplications (e.g., number always ends with a 0, 2, 4, 6 or 8 in the 2X table) and between multiplications (4X table is double the 2X table).

To multiply by 4
17x4=
17x2x2 = 34x2
=68 To multiply by
5 14x5=
(14x10)/2 = 140/2 = 70
**Phase 2: Mental multiplication using partitioning**

- Mental methods for multiplying TU x U can be based on the distributive law of multiplication over addition. This allows the tens and units to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the units can be multiplied first but it is more common to start with the tens.

**Informal recording in Year 4 might be:**

\[
\begin{align*}
40 & \quad +
\
3 & \quad \times 6
\
240 & \quad +
\
18 & \quad = 258
\end{align*}
\]

Also record mental multiplication using partitioning:

\[
\begin{align*}
14 \times 3 & = (10 + 4) \times 3 \\
& = (10 \times 3) + (4 \times 3) = 30 + 12 = 42
\end{align*}
\]

\[
\begin{align*}
43 \times 6 & = (40 + 3) \times 6 \\
& = (40 \times 6) + (3 \times 6) = 240 + 18 = 258
\end{align*}
\]

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

\[
\begin{align*}
7 \times 3 & = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21
\end{align*}
\]

**Phase 3: The grid method**

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

**To multiply 2 digit number by a 1 digit number using the grid method.**

\[
\begin{align*}
38 \times 7 & = (30 \times 7) + (8 \times 7) = 210 + 56 = 266
\end{align*}
\]

**Key Vocabulary**

- product
- multiplication, array, partition,
• The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.

• The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

Phase 4: Expanded short multiplication
• The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

• Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in $38 \times 7$ is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship $3 \times 7$ should be stressed.

• Most children should be able to use this expanded method for TU × U by the end of Year 4.

Phase 5: Short multiplication
• The recording is reduced further, with carry digits recorded below the line.

• If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 4.

Phase 4

\[
\begin{array}{c}
30 + 8 \\
\times 7 \\
\hline
210 \\
56 \\
\hline
266
\end{array}
\]

Phase 5

\[
\begin{array}{c}
38 \\
\times 7 \\
\hline
266
\end{array}
\]

As above

Phase 5

\[
\begin{array}{c}
38 \\
\times 7 \\
\hline
266
\end{array}
\]

As above

Carry.

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
Phase 6: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

56 × 27 is approximately 60 × 30 = 1800.

<table>
<thead>
<tr>
<th></th>
<th>20</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1000</td>
<td>350</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>1512</td>
<td></td>
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<table>
<thead>
<tr>
<th></th>
<th>20</th>
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<tr>
<td>50</td>
<td>6</td>
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<td>×</td>
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<tr>
<td>1000</td>
<td>350</td>
<td>1350</td>
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<tr>
<td>120</td>
<td>42</td>
<td>162</td>
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- Reduce the recording, showing the links to the grid method above.

56 × 27 is approximately 60 × 30 = 1800.

56
× 27
1000 50 × 20 = 1000
120 6 × 20 = 120
350 50 × 7 = 350
42 6 × 7 = 42
1512
1

- Reduce the recording further.
- The carry digits in the partial products of 56 × 20 = 1120 and 56 × 7 = 392 are usually carried mentally.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.

56 × 27 is approximately 60 × 30 = 1800.

56
× 27
1120 56 × 20
392 56 × 7
1512
1

Phase 6: Three-digit by two-digit products

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
- Children may need to do a separate recording when adding two sets of numbers together such as 1600 + 720.

286 × 29 is approximately 300 × 30 = 9000.

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<thead>
<tr>
<th></th>
<th>20</th>
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<tr>
<td>200</td>
<td>4000</td>
<td>1800</td>
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<td>80</td>
<td>1600</td>
<td>720</td>
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<tr>
<td>6</td>
<td>120</td>
<td>54</td>
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<td>8294</td>
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1
• Reduce the recording, showing the links to the grid method above.
• This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

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<tbody>
<tr>
<td>286</td>
<td>29</td>
</tr>
<tr>
<td>4000</td>
<td>200 × 20 = 4000</td>
</tr>
<tr>
<td>1600</td>
<td>80 × 20 = 1600</td>
</tr>
<tr>
<td>120</td>
<td>6 × 20 = 120</td>
</tr>
<tr>
<td>1800</td>
<td>200 × 9 = 1800</td>
</tr>
<tr>
<td>720</td>
<td>80 × 9 = 720</td>
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<tr>
<td>54</td>
<td>6 × 9 = 54</td>
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<td><strong>8294</strong></td>
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**Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.**

• Again, the carry digits in the partial products are usually carried mentally.

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<tbody>
<tr>
<td>286</td>
<td>29</td>
</tr>
<tr>
<td>5720</td>
<td>286 × 20</td>
</tr>
<tr>
<td>2574</td>
<td>286 × 9</td>
</tr>
<tr>
<td><strong>8294</strong></td>
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**Written methods for division**

From phase 2 it shows the building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

• understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
• partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
• recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
• know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
• understand and use multiplication and division as inverse operations.

**Note:** It is important that children’s mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

• understand division as repeated subtraction;
• estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
• multiply a two-digit number by a single-digit number mentally;
• subtract numbers using the column method.
Phase 1

To half numbers to 10/20 – practically and mentally.
To be able to divide practically by sharing.
6 eggs shared between 2 nests = 3

To divide on a number line initially with no remainders and later with remainders.

5 hops in 15. How big is each hop?
$15 \div 5 = 3$
15 shared between 5
Phase 2: Mental division using partitioning

- Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

Phase 2

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining units, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

\[
\begin{align*}
84 & \quad 70 + 14 \\
\downarrow & \quad \downarrow \\
10 & \quad 2 = 12
\end{align*}
\]

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Key Vocabulary

- lots of groups of share half divide division divided by remainder factor quotient
Phase 3: Chunking on a number line and ‘Expanded’ method for HTU ÷ U

- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
- For TU ÷ U there is a link to the mental method.
- As you record the division, ask: ‘How many nines in 90?’ or ‘What is 90 divided by 9?’
- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.
- This method, often referred to as ‘chunking’, is based on subtracting multiples of the divisor, or ‘chunks’. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding children of the link between division and repeated subtraction.
- However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

Phase 3
To divide on a number line by chunking.

137 / 6 = 22 r 5

\[ \begin{array}{cccccc}
0 & 10 & 20 & 30 & 40 & 50 \\
\end{array} \]

\[ \begin{array}{cccccc}
10 & + & 10 & + & 2 & r 5 \\
\end{array} \]

97 ÷ 9

\[ \begin{array}{cccc}
97 & \text{÷} & 9 & \text{Answer:} \\
9 \times 10 & & & 10 \text{ R7} \\
- 90 & \text{÷} & 9 & \text{Answer:} \\
7 & \text{÷} & 9 & 10 \text{ R7} \\
\end{array} \]

6 \text{196}

\[ \begin{array}{cccc}
6 \text{196} & \text{÷} & 6 \times 10 & \text{Answer:} \\
- 60 & \text{÷} & 6 \times 10 & 32 \text{ R 4} \\
& \text{÷} & 6 \times 10 & 4 \text{ R 4} \\
& \text{÷} & 6 \times 2 & \text{Answer:} \\
& \text{÷} & 6 \times 2 & 32 \text{ R 4} \\
& \text{÷} & 6 \times 2 & 4 \text{ R 4} \\
\end{array} \]
• The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that ‘trap’ the HTU dividend.

• Estimating has two purposes when doing a division:
  – to help to choose a starting point for the division;
  – to check the answer after the calculation.

• Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

Phase 5: Short division

A fact or know it box as seen above is used to help the children build up a set of information which can help them with working out multiplication facts of large numbers which will help them solve long and even short division problems. The fact or know it box is a useful tool when the facts needed are not those taught daily such as 30 × 6.

36 × 1 = 36
35 × 2 = 72
35 × 3 = 105
36 × 4 = 144
35 × 5 = 180
36 × 6 = 216

To calculate the answers to 36 × 6 a child may resort back to partitioning which is mentioned elsewhere and use this knowledge to help them calculate the unusual multiplication facts required to solve the answers to a difficult division or multiplication problem.

A fact or know it box will probably be used at this phase to help complete the calculation. Children should be able to choose whichever method they are most comfortable with to help with confidence and enable them to move forward.

Standard short division method

\[
\begin{array}{c|c}
6 & 196 \\
\hline
-180 & 6 \times 30 \\
16 & 6 \times 2 \\
\hline
4 & 32 \\
\end{array}
\]

Answer: 32 R 4

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Phase 6: Long division

To calculate 748 divided by 51,

First, set the sum out as shown:

\[
\begin{array}{c|c}
51 & 748 \\
\end{array}
\]
We work out 74 divided by 51, and write the answer (1) above the 4.
1 × 51 = 51, so we write this underneath 74.
Subtract 51 from 74 to get the remainder (23).

\[
\begin{array}{c}
51 \overline{748} \\
-51 \\
\hline
23
\end{array}
\]

We now bring down the next digit (8) and write it on the end of the 23. This is the same as writing the remainder at the top.

\[
\begin{array}{c}
51 \overline{748} \\
-51 \\
\hline
238
\end{array}
\]

We now work out 238 divided by 51, and write the answer (4) above the 8. You use estimation skills here: 51 is roughly 50 and 4 × 50 = 200. You can work out 51 × 4 = 204 separately. We write 204 underneath the 238 and subtract to find the remainder. There are no more digits to bring down, so we have our answer:

\[
\begin{array}{c}
51 \overline{748} \\
-51 \\
\hline
238 \\
-204 \\
\hline
34
\end{array}
\]

So the answer is 14 remainder 34.

Another method we use can be seen below this is a version of short division as is easier for the children to follow and they generally make less mistakes with this method. However children must be taught the underlying process of what how may means in order to understand this method.

\[
\begin{array}{c}
15 \overline{346} \\
3 \overline{46} \\
\hline
023 \text{ r1}
\end{array}
\]

Know It:
1 = 15
2 = 30
4 = 60
8 = 120
10 = 150
5 = 75

How many 15s go into 3? 0 remainder 3
How many 15s go into 34? 2 remainder 4
How many 15s go into 46? 3 remainder 1
Children working at phase 6 should also be expected to:

- solve division problems involving measures and money.
- use as an inverse operation to check multiplication calculations.
- convert remainders to decimal remainders.

Children at phase 6 and beyond would also be expected to tackle division problems which involve decimals. Presenting their answers in 3 ways e.g. remainder, fraction, decimal.
Place Value

Place value in an extremely important concept that is taught early in a child’s education and as students learn about larger numbers, the concept of place value continues throughout KS 2. Place value refers to the value of the digit based on its position. Place value is a concept that is difficult for young learners.

Think about the way young learners learn about numbers. They begin with rote counting, 1, 2, 3, 4... From there they get to two digit numbers, 11, 12, 13 and to three digit numbers 100, 101, 102... To a child the 1 in 1, 10 and 100 often mean the same thing. However, in place value, a 1 is one, a 10 is 1 group of ten, 100, is ten, tens or 1 group of 100. Therefore, the difficulty is understanding the place of the specific number and knowing that the placement changes the value of the digit.

It is vital that children have an excellent understanding of place value and how it works in order for them to understand and use other numerical concepts.

Place value understanding needs to be taught and revisited across all year groups to ensure all children have a solid grasp of the concept.